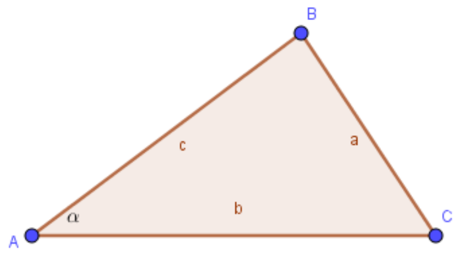
Buktikan :

$$LVABC = \sqrt{s(s-a)(s-b)(s-c)},\ dengan\ s = \tfrac{1}{2}(a+b+c)$$

a, b, c *sisi segitiga ABC.*

Bukti : *misalkan* $\alpha = \angle(BAC)$



$$i)\ \sin^2\alpha + \cos^2\alpha = 1 \quad (identitas)$$

$$\sin^2\alpha = 1 - \cos^2\alpha$$

$$\sin^2\alpha = (1-\cos\alpha)(1+\cos\alpha)$$

$$\sin^2\alpha = \left(1 - \frac{b^2+c^2-a^2}{2bc}\right)\left(1 + \frac{b^2+c^2-a^2}{2bc}\right)$$

$$\sin^2\alpha = \left(\frac{2bc-b^2-c^2+a^2}{2bc}\right)\left(\frac{2bc+b^2+c^2-a^2}{2bc}\right)$$

$$\sin^2\alpha = \left(\frac{-(b^2+c^2-2bc)+a^2}{2bc}\right)\left(\frac{2bc+b^2+c^2-a^2}{2bc}\right)$$

$$\sin^2\alpha = \left(\frac{a^2-(b-c)^2}{2bc}\right)\left(\frac{(b+c)^2-a^2}{2bc}\right)$$

$$\sin^2\alpha = \left(\frac{(a-b+c)(a+b-c)(b+c-a)(b+c+a)}{4b^2c^2}\right)$$

$$\sin^2\alpha = \left(\frac{2(s-b).2(s-c).2(s-a)(2s)}{4b^2c^2}\right)$$

$$\sin\alpha = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$$

dengan :

$$a+b+c = 2s$$

$$b+c-a = a+b+c-2a = 2s-2a = 2(s-a)$$

$$a+c-b = a+b+c-2b = 2s-2b = 2(s-b)$$

$$a+b-c = a+b+c-2c = 2s-2c = 2(s-c)$$

$$ii)\ LVABC = \frac{1}{2}bc.\sin\alpha,\ \alpha = \angle A$$

$$LVABC = \frac{1}{2}bc\frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$$

$$LVABC = \sqrt{s(s-a)(s-b)(s-c)}\ (terbukti)$$

Note.

$$i)\ \cos\alpha = \frac{b^2+c^2-a^2}{2bc} \quad (aturan\ cosinus)$$

$$ii)\ (b+c)^2 = b^2+c^2+2bc$$

$$(b-c)^2 = b^2+c^2-2bc$$